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1999 J. Phys.: Condens. Matter 11 4675

(http://iopscience.iop.org/0953-8984/11/24/309)

Interband resonant magnetotunnelling in semiconductor heterostructures

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Received 6 January 1999, in final form 19 March 1999

Abstract. The interband resonant magnetotunnelling processes in double-barrier structures made from A_3B_5 materials in a magnetic field normal to the interfaces are investigated theoretically. The multiband Kane model and transfer-matrix method are used to obtain the wavefunctions of particles in a heterostructure taking into account the mixing at the interfaces of the states corresponding to different Landau levels due to the spin–orbit interaction. The dependencies of the transmission coefficients for the transitions between the states of the various Landau levels on the incident particle energy and the dependencies of the tunnelling current density on the voltage for different values of magnetic field are investigated. It is shown that the interband transitions between the states with different Landau-level indices result in the additional peaks of the theoretical current–voltage characteristics of resonant tunnelling structures (RTS) such as the InAs/AlGaSb/GaSb RTS.

1. Introduction

In the resonant tunnelling structures (RTS) with type II heterojunctions such as the InAs/AlSb/GaSb RTS with a GaSb quantum well and the GaSb/AlSb/InAs RTS with an InAs quantum well, the interband tunnelling of electrons through the quasibound states in the valence band quantum well, or holes from GaSb through the quasibound states in the conduction band quantum well, occurs. These interband RTS show fairly high values of the peak-to-valley current ratio at room temperature and have attracted considerable attention from researchers (see, for example, references [1–17]). In papers [3–5, 7] the interband magnetotunnelling in RTS made from InAs, AlSb, and GaSb was investigated experimentally. In a strong magnetic field normal to the interfaces, interband tunnelling current oscillations versus voltage were observed, conditioned by the interband resonant tunnelling through different Landau levels [3, 4]. Applying a magnetic field parallel to the interfaces results in a considerable shift in the peak voltage [7]. The interband resonant magnetotunnelling with the magnetic field parallel to the interfaces was considered theoretically in reference [15] using the eight-band model. In reference [14] the current–voltage (I–V) characteristics of interband RTS with the magnetic field parallel to the current were calculated using the simplified two-band model. The authors of reference [15] neglected the quantization of the particle spectrum in a magnetic field. In reference [14] this effect was taken into account, but the effect of inter-Landau-level transitions was not considered. The aim of this paper is to investigate theoretically the interband resonant magnetotunnelling in heterostructures in a magnetic field normal to the interfaces, taking these effects into account.

The transmission coefficients and tunnelling current-density components corresponding to the processes of interband tunnelling from the states of each Landau level, with conservation

0953-8984/99/244675+14$19.50

and with change of the Landau-level index n, were calculated using the eight-band Kane model [18] and the transfer-matrix method [19, 21]. The latter processes can occur without scattering because of the mixing of the states corresponding to different indices n at the interfaces due to the spin–orbit interaction [22]. We show that in the InAs/AlGaSb/GaSb RTS the current density due to the interband transitions with a changing Landau-level index may be comparable with the current density due to the interband transitions with conservation of the Landau-level index, which leads to the additional peaks in the theoretical I–V characteristics of these RTS.

The inter-Landau-level tunnelling processes of electrons or holes were observed experimentally, when investigating the peculiarities of the I–V characteristics of the structures with intraband tunnelling mechanisms [23–27]. These peculiarities (the oscillations of the I–V characteristics or the oscillations of the corresponding second derivatives $\mathrm{d}^2I/\mathrm{d}V^2$ versus voltage) for values of the external bias higher than that corresponding to the main peak of the current density were explained as arising due to inter-Landau-level elastic- and inelastic-scattering-assisted tunnelling processes. The authors of references [3, 4] explained the oscillations of the I–V characteristics of the interband RTS as arising due to intra-Landau-level tunnelling processes. They associated the small additional peak at 15 T with circuit instabilities. We believe that some of the additional peaks of the current or second derivative $\mathrm{d}^2I/\mathrm{d}V^2$ may be associated with the tunnelling processes with a changing Landau-level index without scattering.

2. Model description

We use the $\boldsymbol{k}\cdot\boldsymbol{p}$ method and take into account the coupling of the conduction band with the light- and heavy-hole bands and with the split-off band exactly, and neglect the higher bands, to investigate the interband magnetotunnelling processes in RTS such as an InAs/AlGaSb/GaSb RTS, whose conduction and valence band diagram is shown in figure 1. In this way we consider only the processes of interband tunnelling through the light-hole states in the quantum well, which are dominant for the values of the external bias for which these processes can occur [11, 12, 16]. We neglect the phonon-assisted tunnelling, which can govern the total interband tunnelling current density in structures with a wide-gap barrier layer thickness greater than 100 Å [28]. If the z-axis is normal to the interfaces, then the 8×8 Hamiltonian can be

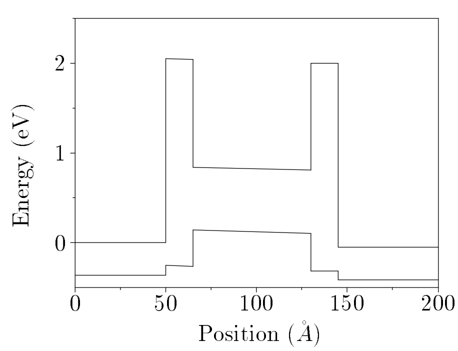

Figure 1. The conduction and valence band diagram of the InAs/AlGaSb/GaSb RTS.

written as

$$\hat{H} = \begin{pmatrix} \hat{H}_{+-} & \hat{H}_{--} \\ \hat{H}_{++} & \hat{H}_{-+} \end{pmatrix} \tag{1}$$

where

$$\hat{H}_{\pm\mp} = \begin{pmatrix} E_C(z) & \sqrt{2}\mathrm{i}P\hat{k}_z/\sqrt{3} & -\mathrm{i}P\hat{k}_z/\sqrt{3} & P\hat{k}_\pm \\ -\sqrt{2}\mathrm{i}P\hat{k}_z/\sqrt{3} & E_V(z) & 0 & 0 \\ \mathrm{i}P\hat{k}_z/\sqrt{3} & 0 & E_V(z) - \Delta(z) & 0 \\ P\hat{k}_\mp & 0 & 0 & E_V(z) \end{pmatrix} \tag{2}$$

and

$$\hat{H}_{\pm\pm} = \begin{pmatrix} 0 & P\hat{k}_\pm/\sqrt{3} & \sqrt{2}P\hat{k}_\pm/\sqrt{3} & 0 \\ P\hat{k}_\pm/\sqrt{3} & 0 & 0 & 0 \\ \sqrt{2}P\hat{k}_\pm/\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3}$$

In equations (2), (3), $\hat{k}_\pm = \mp\mathrm{i}(\hat{k}_x \pm \mathrm{i}\hat{\bar{k}}_y)/\sqrt{2}$, $\hat{k}_x = -\mathrm{i}\,\partial/\partial x$, $\hat{\bar{k}}_y = -\mathrm{i}\,\partial/\partial y + |e|Bx/(\hbar c)$, $\hat{k}_z = -\mathrm{i}\,\partial/\partial z$, e is the electron charge, B is the magnetic field, c is the light velocity, $E_C(z)$ and $E_V(z)$ are the conduction and valence band edges, $\Delta(z)$ is the split-off energy, and the value of $P = -\hbar^2\langle S|\partial/\partial z|Z\rangle/m_0$, where $|S\rangle$, $|Z\rangle$ are the states corresponding to the conduction and valence bands and m_0 is the free-electron mass, is proportional to the interband momentum matrix element. We have assumed that the magnetic field $\boldsymbol{B}$ is parallel to the z-axis, so $B_z = B$, $B_x = 0$, $B_y = 0$; and the components of vector potential are: $A_y = Bx$, $A_x = A_z = 0$. We use the same basis functions as in reference [29]; these are listed in the appendix. The basis functions and the value of P are assumed to be the same throughout the whole structure. Also, we neglected the g-factor of the free electron and the term $\hbar^2\hat{\boldsymbol{k}}^2/(2m_0)$ in the expression for $\hat{H}_{ii}$, as in reference [30], which is possible because the electron effective mass is much less than m_0. The existence of these terms in the Hamiltonian leads to spurious unphysical solutions of the secular equation [31–34], which are now removed. Thus the envelope functions ψ_i obey the equations

$$\sum \hat{H}_{ij}\psi_j = \bar{E}\psi_i \qquad i = 1, 2, \ldots, 8. \tag{4}$$

In (4) ψ_i is an envelope function and $\bar{E}$ is the energy. The boundary conditions in the envelope function approach can be derived directly from the equations for the envelope functions with position-dependent parameters by means of integration of these equations across an interface [20, 34–38]. Suppose that equations (4) with the Hamiltonian given by (1)–(3), where $E_g(z) = E_C(z) - E_V(z)$ and $\Delta(z)$ are piecewise-constant functions, have finite solutions for ψ_i throughout the whole structure including the regions containing the abrupt heterojunctions. Then integrating each equation of system (4) from $z_j - 0$ to $z_j + 0$, where z_j is the interface coordinate, and taking into account the fact that P is a constant value, we obtain the result that at the interfaces the following functions are continuous:

$$\psi_1 \qquad \sqrt{2}\psi_2 - \psi_3 \qquad \psi_5 \qquad \sqrt{2}\psi_6 - \psi_7. \tag{5}$$

Since the spurious solutions are removed by means of modification of the Hamiltonian as, for example, in references [31, 34], the number of boundary conditions is less than the number of envelope functions and their first derivatives with respect to z. These boundary conditions provide the conservation of the probability current-density component normal to the interfaces for the solutions of equation (4) with the Hamiltonian given by (1)–(3), as is shown in section 3.

The equation system (4) for a bulk material in the case of the absence of an electric field has two solutions for a given value of the energy $\bar{E}$ and Landau-level index n with wave-vector components $k_z = k_z^{\pm}$ and opposite average values of the spin. For the first solution, $\psi_1 \neq 0$, $\psi_5 = 0$, while for the second solution, $\psi_5 \neq 0$, $\psi_1 = 0$ [30]. In the conduction band these two solutions correspond to the states with spin σ approximately equal to $\pm 1/2$, respectively. The dispersions $E_n(k_z)$, where $E_n = \bar{E} - E_V$, are given by

$$\frac{E_n(E_n - E_g)(E_n + \Delta)}{P^2 s(E_n + 2\Delta/3)} - \frac{k_z^{\pm 2}}{s} \pm \frac{\Delta/3}{E_n + 2\Delta/3} = 2n + 1 \qquad n = 0, 1, \ldots. \tag{6}$$

In (6), $s = |e|B/(\hbar c)$. The corresponding envelope functions can be written as

$$\psi_i = \phi_i(x') \exp(\mathrm{i}k_y y + \mathrm{i}k_z z) \tag{7}$$

where $x' = x - x_0$ $(x_0 = -k_y/s)$. For the first solution,

$$\begin{aligned}
&\phi_1 = f_n && \phi_2 = -\frac{\mathrm{i}\sqrt{2}Pk_z^+}{\sqrt{3}E_n} f_n && \phi_3 = \frac{\mathrm{i}Pk_z^+}{\sqrt{3}(E_n + \Delta)} f_n \\
&\phi_4 = \frac{2nP\sqrt{s}}{\sqrt{2}E_n} f_{n-1} && \phi_5 = 0 && \phi_6 = \frac{P\sqrt{s}}{\sqrt{6}E_n} f_{n+1} \\
&\phi_7 = \frac{P\sqrt{s}}{\sqrt{3}(E_n + \Delta)} f_{n+1} && \phi_8 = 0. &&
\end{aligned} \tag{8}$$

Here

$$f_n(x') = \begin{cases} \exp(-sx'^2/2) H_n(\sqrt{s}x') & n \geqslant 0 \\ 0 & n < 0 \end{cases} \tag{9}$$

where $H_n(t)$ is the Hermite polynomial. For the second solution,

$$\begin{aligned}
&\phi_1 = 0 && \phi_2 = \frac{2nP\sqrt{s}}{\sqrt{6}E_n} f_{n-1} && \phi_3 = \frac{2nP\sqrt{s}}{\sqrt{3}(E_n + \Delta)} f_{n-1} \\
&\phi_4 = 0 && \phi_5 = f_n && \phi_6 = -\frac{\mathrm{i}\sqrt{2}Pk_z^-}{\sqrt{3}E_n} f_n \\
&\phi_7 = \frac{\mathrm{i}Pk_z^-}{\sqrt{3}(E_n + \Delta)} f_n && \phi_8 = \frac{P\sqrt{s}}{\sqrt{2}E_n} f_{n+1}. &&
\end{aligned} \tag{10}$$

Near the conduction band edge, the nonparabolicity can be neglected. Then the envelopes ψ_1 and ψ_5, which correspond to the s-type basis functions of the conduction band, are the solutions of the Schrödinger equation for the values of spin $\pm 1/2$, respectively, in the case of the simple effective-mass approximation. Therefore the Landau-level index for the states in the conduction band with spin approximately equal to 1/2 coincides with the index n of the function f_n which defines the function ϕ_1, while the Landau-level index for the states in the conduction band with spin approximately equal to $-1/2$ coincides with the index n of the function f_n which defines the function ϕ_5. Note that equations (6) have spurious solutions if $E_n \approx -2\Delta/3$. These energy levels are significantly lower than the energy interval in which the interband tunnelling can occur in the structures under consideration, so these solutions are not taken into account.

The solutions of the Schrödinger equation in the RTS and the transmission coefficients for the transitions between the states corresponding to various Landau-level indices can be obtained using the solutions for the wavefunctions in bulk materials defined by (7)–(10) and the conditions of continuity at the interfaces of functions (5). If the split-off energy $\Delta(z)$ is equal to zero, then, for the first solution of the Schrödinger equation for a given index n,

$$\psi_1 \neq 0 \qquad \sqrt{2}\psi_2 - \psi_3 \neq 0 \qquad \psi_5 = 0 \qquad \sqrt{2}\psi_6 - \psi_7 = 0.$$

For the second solution of this equation,

$$\psi_1 = 0 \qquad \sqrt{2}\psi_2 - \psi_3 = 0 \qquad \psi_5 \neq 0 \qquad \sqrt{2}\psi_6 - \psi_7 \neq 0.$$

Then, for a heterostructure for two bulk states with opposite values of k_z corresponding to Landau-level index n and spin 1/2 or $-1/2$, we have only two boundary conditions, which can be satisfied without mixing the states of different Landau levels or states with different spin orientations. If $\Delta(z) \neq 0$, then, for the first solution of the Schrödinger equation, $\sqrt{2}\psi_6 - \psi_7 \neq 0$. For the second solution of the Schrödinger equation, $\sqrt{2}\psi_2 - \psi_3 \neq 0$ for $n > 0$. Since more than two functions (5) are not equal to zero, the boundary conditions cannot be satisfied without accounting for the mixing effects. The functions ψ_2 and ψ_3 (ψ_6 and ψ_7) of level n with spin along the magnetic field direction and level $n+1$ with spin opposite to the magnetic field direction are both proportional to f_n (f_{n+1}). Hence in the heterostructure, due to the spin–orbit interaction, the mixing at the interfaces of the states with opposite spin orientations of Landau levels n and $n+1$ occurs in accordance with the boundary conditions. Hence an electron from the state corresponding to Landau-level index n with spin along the direction of the magnetic field can tunnel not only into a similar state to the right of the RTS, but also into the state corresponding to Landau-level index $n+1$ with spin opposite to the direction of the magnetic field. Only the states with Landau-level index $n = 0$ and spin opposite to the direction of the magnetic field are not mixed with other states.

Note that in the framework of the simple effective-mass approximation the term in the Hamiltonian which describes the spin–orbit interaction is usually neglected. In this case the mixing of the states with different Landau-level indices in a magnetic field normal to the interfaces does not exist. For this reason the inter-Landau-level processes with energy conservation in the RTS with an intraband tunnelling mechanism were explained by elastic-scattering-assisted processes. In fact, these processes can occur without scattering. Note also that the authors of reference [39] believe that in the case of an in-plane magnetic field the inter-Landau-level tunnelling processes without scattering are permitted even in a simple one-band model in which the spin–orbit interaction is neglected.

3. The interband resonant tunnelling probability and tunnelling current density

The transfer-matrix method [19–21] is an appropriate approach for investigating the interband resonant tunnelling in structures under external bias in the presence of a magnetic field. In this case the real potential distribution is replaced by a stepwise-constant one. We calculated the transfer matrices to obtain the transmission coefficients for the transitions from the states with different spin orientations corresponding to each value of the Landau-level index. For the state with $n = 0$ and spin opposite to the direction of the magnetic field, the envelope function ψ_5, which is a combination of incident and reflected waves with pre-exponent coefficients A_2^j and B_2^j, respectively, in the sublayer j at $z = z_j$, is given by

$$\psi_5 = A_2^j f_0(x') \exp(\mathrm{i}k_y y + \mathrm{i}k_{z_j} z_j) + B_2^j f_0(x') \exp(\mathrm{i}k_y y - \mathrm{i}k_{z_j} z_j). \tag{11}$$

The other envelopes can easily be obtained from equations (6), (7), (10). Then the coefficients A_2^j, B_2^j and A_2^{j+1}, B_2^{j+1} for the adjacent sublayers j and $j+1$ obey the equation

$$\begin{pmatrix} A_2^j \\ B_2^j \end{pmatrix} = M_j \begin{pmatrix} A_2^{j+1} \\ B_2^{j+1} \end{pmatrix}. \tag{12}$$

The transfer matrix M_j at $z = z_j$ can be written as

$$M_j^{\pm} = \frac{1}{2}\begin{pmatrix} [1 + k_{z_{j+1}} P_{j+1}/(k_{z_j} P_j)]\exp(-\mathrm{i}b_j z_j) & [1 - k_{z_{j+1}} P_{j+1}/(k_{z_j} P_j)]\exp(-\mathrm{i}a_j z_j) \\ [1 - k_{z_{j+1}} P_{j+1}/(k_{z_j} P_j)]\exp(\mathrm{i}a_j z_j) & [1 + k_{z_{j+1}} P_{j+1}/(k_{z_j} P_j)]\exp(\mathrm{i}b_j z_j) \end{pmatrix} \tag{13}$$

where

$$a_j = k_{z_j} + k_{z_{j+1}} \qquad b_j = k_{z_j} - k_{z_{j+1}} \tag{14}$$

$$P_j = 2/E(z_j) + 1/(E(z_j) + \Delta(z_j)). \tag{15}$$

In the latter equation, $E(z_j) = \bar{E} - E_V(z_j)$. Then the amplitudes of the incident, reflected, and transmitted waves A_2^0, B_2^0, and A_2^N are given by

$$\begin{pmatrix} A_2^0 \\ B_2^0 \end{pmatrix} = \prod_{j=0}^{N-1} M_j \begin{pmatrix} A_2^N \\ 0 \end{pmatrix} \tag{16}$$

where N is the number of steps in the calculation.

If $n \neq 0$ or the spin of the tunnelling electron is along the direction of the magnetic field, then the solution of the Schrödinger equation can be composed of the superposition of the states for which $\psi_1 \neq 0$ and $\psi_5 \neq 0$ corresponding to the values of the Landau-level index of n and $n + 1$, respectively. For this solution

$$\psi_1 = A_1^j f_n(x') \exp(\mathrm{i}k_y y + \mathrm{i}k_{z_j}^+ z_j) + B_1^j f_n(x') \exp(\mathrm{i}k_y y - \mathrm{i}k_{z_j}^+ z_j) \tag{17}$$

$$\psi_5 = A_2^j f_{n+1}(x') \exp(\mathrm{i}k_y y + \mathrm{i}k_{z_j}^- z_j) + B_2^j f_{n+1}(x') \exp(\mathrm{i}k_y y - \mathrm{i}k_{z_j}^- z_j). \tag{18}$$

The other envelope functions for a given value of the incident particle energy can be obtained taking into account (6)–(10). Then the coefficients $A_{1,2}^j$, $B_{1,2}^j$ can be obtained from the coefficients $A_{1,2}^{j+1}$, $B_{1,2}^{j+1}$ using a 4×4 transfer matrix M_j in the following way:

$$\begin{pmatrix} A_1^j \\ B_1^j \\ A_2^j \\ B_2^j \end{pmatrix} = M_j \begin{pmatrix} A_1^{j+1} \\ B_1^{j+1} \\ A_2^{j+1} \\ B_2^{j+1} \end{pmatrix}. \tag{19}$$

The elements of the 4×4 transfer matrix can be written as

$$\begin{aligned}
M_{j11} &= [1 + k_{z_{j+1}}^+ P_{j+1}/(k_{z_j}^+ P_j)]\exp(-\mathrm{i}b_j^+ z_j)/2 \\
M_{j12} &= [1 - k_{z_{j+1}}^+ P_{j+1}/(k_{z_j}^+ P_j)]\exp(-\mathrm{i}a_j^+ z_j)/2 \\
M_{j13} &= [-\mathrm{i}\sqrt{s}(n+1)(Q_j - Q_{j+1})/(k_{z_j}^+ P_j)]\exp(-\mathrm{i}c_j^- z_j) \\
M_{j14} &= [-\mathrm{i}\sqrt{s}(n+1)(Q_j - Q_{j+1})/(k_{z_j}^+ P_j)]\exp(-\mathrm{i}c_j^+ z_j) \\
M_{j21} &= [1 - k_{z_{j+1}}^+ P_{j+1}/(k_{z_j}^+ P_j)]\exp(\mathrm{i}a_j^+ z_j)/2 \\
M_{j22} &= [1 + k_{z_{j+1}}^+ P_{j+1}/(k_{z_j}^+ P_j)]\exp(\mathrm{i}b_j^+ z_j)/2 \\
M_{j23} &= [\mathrm{i}\sqrt{s}(n+1)(Q_j - Q_{j+1})/(k_{z_j}^+ P_j)]\exp(\mathrm{i}c_j^+ z_j) \\
M_{j24} &= [\mathrm{i}\sqrt{s}(n+1)(Q_j - Q_{j+1})/(k_{z_j}^+ P_j)]\exp(\mathrm{i}c_j^- z_j)
\end{aligned} \tag{20a}$$

$$
\begin{aligned}
M_{j31} &= [-\mathrm{i}\sqrt{s}(Q_j - Q_{j+1})/(2k_{z_j}^- P_j)] \exp(-\mathrm{i}d_j^- z_j) \\
M_{j32} &= [-\mathrm{i}\sqrt{s}(Q_j - Q_{j+1})/(2k_{z_j}^- P_j)] \exp(-\mathrm{i}d_j^+ z_j) \\
M_{j33} &= [1 + k_{z_{j+1}}^- P_{j+1}/(k_{z_j}^- P_j)] \exp(-\mathrm{i}b_j^- z_j)/2 \\
M_{j34} &= [1 - k_{z_{j+1}}^- P_{j+1}/(k_{z_j}^- P_j)] \exp(-\mathrm{i}a_j^- z_j)/2 \\
M_{j41} &= [\mathrm{i}\sqrt{s}(Q_j - Q_{j+1})/(2k_{z_j}^- P_j)] \exp(\mathrm{i}d_j^+ z_j) \\
M_{j42} &= [\mathrm{i}\sqrt{s}(Q_j - Q_{j+1})/(2k_{z_j}^- P_j)] \exp(\mathrm{i}d_j^- z_j) \\
M_{j43} &= [1 - k_{z_{j+1}}^- P_{j+1}/(k_{z_j}^- P_j)] \exp(\mathrm{i}a_j^- z_j)/2 \\
M_{j44} &= [1 + k_{z_{j+1}}^- P_{j+1}/(k_{z_j}^- P_j)] \exp(\mathrm{i}b_j^- z_j)/2.
\end{aligned}
\tag{20b}
$$

In equations (20),

$$
\begin{aligned}
a_j^\pm &= k_{z_j}^\pm + k_{z_{j+1}}^\pm \qquad & b_j^\pm &= k_{z_j}^\pm - k_{z_{j+1}}^\pm \\
c_j^\pm &= k_{z_j}^+ \pm k_{z_{j+1}}^- \qquad & d_j^\pm &= k_{z_j}^- \pm k_{z_{j+1}}^+
\end{aligned}
\tag{21}
$$

$$
Q_j = 1/E(z_j) - 1/(E(z_j) + \Delta(z_j)). \tag{22}
$$

Then the amplitudes of the incident, reflected, and transmitted waves $A_{1,2}^0$, $B_{1,2}^0$, and $A_{1,2}^N$ can be obtained from the following equations:

$$
\begin{pmatrix} A_1^0 \\ B_1^0 \\ A_2^0 \\ B_2^0 \end{pmatrix} = \prod_{j=0}^{N-1} M_j \begin{pmatrix} A_1^N \\ 0 \\ A_2^N \\ 0 \end{pmatrix}. \tag{23}
$$

The elements M_{j13}, M_{j14}, M_{j23}, M_{j24}, M_{j31}, M_{j41}, M_{j32}, M_{j42} of the transfer matrix, equations (20), describe the mixing of the states of different Landau-level indices. Each of these elements is proportional to $\sqrt{B}$. Therefore the probability of inter-Landau-level transitions is considerable only in sufficiently strong magnetic fields. Each of these elements of the transfer matrix, equations (20), is equal to zero if $\Delta(z) = 0$, because in accordance with (22) Q_j and Q_{j+1} are equal to zero. Hence the probability of inter-Landau-level transitions increases with the spin–orbit interaction increasing. In the case of an electron tunnelling between bands in a RTS with type I heterojunctions, if $|E(z)| \gg \Delta(z)$, the probability of inter-Landau-level transitions is negligible. For example, in the GaAs/AlGaAs RTS with contacts doped by donors, the intra-Landau-level transitions of conduction band electrons are dominant. In the case of an electron tunnelling between bands in a RTS with type II heterojunctions, such as the InAs/AlGaSb/GaSb RTS, if $|E(z)| \ll \Delta(z)$ in the barrier, the probability of inter-Landau-level transitions is considerable in sufficiently strong magnetic fields.

The equation for the probability-flux-density component normal to the interfaces, j_z, in the presence of a magnetic field, which is used to calculate the transmission coefficients, is obtained in a conventional way (see, for example, reference [40]) from the equation

$$
\frac{\partial}{\partial t} \sum_i |\psi_i|^2 + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0. \tag{24}
$$

Using the time-dependent equations for envelope functions, we obtain

$$
\frac{\partial}{\partial t} \sum_i |\psi_i|^2 = \frac{\mathrm{i}}{\hbar} \sum_{ij} (\psi_i \hat{H}_{ij}^* \psi_j^* - \psi_i^* \hat{H}_{ij} \psi_j). \tag{25}
$$

Then the equation for j_z is derived using (1)–(3), (24), (25) and can be written as

$$
j_z = \frac{\mathrm{i}P}{\hbar\sqrt{3}} [\psi_1(\sqrt{2}\psi_2^* - \psi_3^*) - \psi_1^*(\sqrt{2}\psi_2 - \psi_3) + \psi_5(\sqrt{2}\psi_6^* - \psi_7^*) - \psi_5^*(\sqrt{2}\psi_6 - \psi_7)]. \tag{26}
$$

Taking into account the boundary conditions, we obtain that j_z is continuous at the interfaces. Then the coefficients of transmission for transitions between the different states can be obtained as the ratios of the corresponding probability flux densities of the interfaces averaged over the coordinate x for the transmitted and incident waves $\langle j_{zk}^N \rangle$ and $\langle j_{zl}^0 \rangle$, and are given by

$$T_{kl} = \langle j_{zk}^N \rangle / \langle j_{zl}^0 \rangle \qquad k, l = 1, 2 \tag{27}$$

where k $(l) = 1$ corresponds to the wave with spin along the direction of the magnetic field and Landau-level index n, k $(l) = 2$ corresponds to the opposite spin orientation and Landau-level index $n + 1$ for $n \geqslant 0$. If for the incident wave with spin opposite to the direction of the magnetic field the Landau-level index is equal to zero, then only the coefficient $T_{22} \neq 0$. For the quantities $\langle j_{z1}^{0,N} \rangle$, $\langle j_{z2}^{0,N} \rangle$ corresponding to the Landau-level indices n and $n+1$, respectively, we obtain

$$\begin{aligned} \langle j_{z1}^{0,N} \rangle &\sim 2^n n! |A_1^{0,N}|^2 k_{z_{0,N}}^+ P_{0,N} \\ \langle j_{z2}^{0,N} \rangle &\sim 2^{n+1} (n+1)! |A_2^{0,N}|^2 k_{z_{0,N}}^- P_{0,N} \end{aligned} \tag{28}$$

where the amplitudes of the waves $|A_{1,2}^{0,N}|$ are given by (23) for $n \geqslant 0$. If $n + 1 = 0$, then the amplitudes of the envelope functions for the states with zero Landau-level index and spin opposite to the direction of the magnetic field, $|A_2^{0,N}|$, are given by (16).

Using the transmission coefficients T_{kl}, the total interband tunnelling current density, which is the sum of the different interband tunnelling current components, can be calculated in the following way:

$$j = \frac{|e| L_y L_z}{V (2\pi)^2 \hbar} \sum_{k,l,n} \int \mathrm{d}k_y \, \mathrm{d}k_z \, T_{kl} (f_1 - f_2) \frac{\partial E_n}{\partial k_z}. \tag{29}$$

In this equation, $V = L_x L_y L_z$, L_i is a normalizing length, $E_n(k_z)$ is the dispersion law for electrons in the left-hand contact layer, f_1, f_2 are the occupations of the states to the left and to the right of the tunnelling structure, respectively. The total current density is composed of the current-density components corresponding to the transitions between the initial and final states with various values of Landau-level index and various spin orientations. Taking into account that

$$\int \mathrm{d}k_y = s \int_{-L_x/2}^{L_x/2} \mathrm{d}x_0 = L_x s \tag{30}$$

we obtain for zero temperature and positive external bias V

$$j = \frac{|e| s}{(2\pi)^2 \hbar} \sum_{k,l,n} \int_{\max(0, E_{F2})}^{E_{F1}} \mathrm{d}E_n \, T_{kl} \tag{31}$$

where E_{F1}, E_{F2} are the Fermi levels to the left and to the right of the tunnelling structure, respectively. For a symmetrical structure, $E_{F1} = E_F$, $E_{F2} = E_F - |e|V$, where the value of E_F is related to the electron concentration N_e in the contacts in the following way:

$$N_e = \frac{s}{(2\pi)^2} \sum_{n,\sigma} \int_0^{E_F} \mathrm{d}E_n \, \frac{\partial k_z}{\partial E_n}. \tag{32}$$

Here the values of σ correspond to two different spin orientations. We investigated the peculiarities of the I–V characteristics of a RTS with an interband tunnelling mechanism using equations (31), (32), and the formulae derived for the transmission coefficient calculation taking into account the quantization of the particle spectrum in a magnetic field and inter-Landau-level transitions.

4. Results and discussion

The results of the calculation of the transmission coefficients versus incident particle energy $E = \bar{E} - E_C$ for the InAs/AlGaSb/GaSb RTS with two InAs contact layers, two 25 Å AlGaSb barriers, and a 70 Å GaSb quantum well in a magnetic field of 15 T for the values of external bias 0.03, 0.05, and 0.07 V are shown in figures 2(a), 2(b), and 2(c), respectively. We have used the same parameters as in reference [29]. Curves 1 in these figures correspond to the transitions from the electron states to the left of the double-barrier structure with spin opposite to the direction of the magnetic field corresponding to the value $n = 0$ into similar states to the right of it. Curves 2 represent the transmission coefficients versus energy for the transitions from the states in the conduction band of the left-hand InAs layer corresponding to the Landau-level index $n = 0$ and spin along the direction of the magnetic field into similar states to the right of the tunnelling structure. Curves 3 represent the transmission coefficients versus energy for the transitions from the states in the conduction band of the left-hand InAs layer with the Landau-level index $n = 0$ and spin along the direction of the magnetic field into the states to the right of the double-barrier structure with Landau-level index $n = 1$ and opposite spin orientation. In all of these processes the resonant interband tunnelling occurs through the light-hole states in the valence band quantum well. The calculations showed that the transition probability for the processes with the changing Landau-level index can be comparable with the transition probability for the processes with conservation of the Landau-level index. Due to strong mixing of the quasibound states in the valence band quantum well corresponding to the values $n = 0$ and $n = 1$, resonant tunnelling in the cases of curves 2 and 3 occurs through the two states in the quantum well with different spin orientations, which correspond

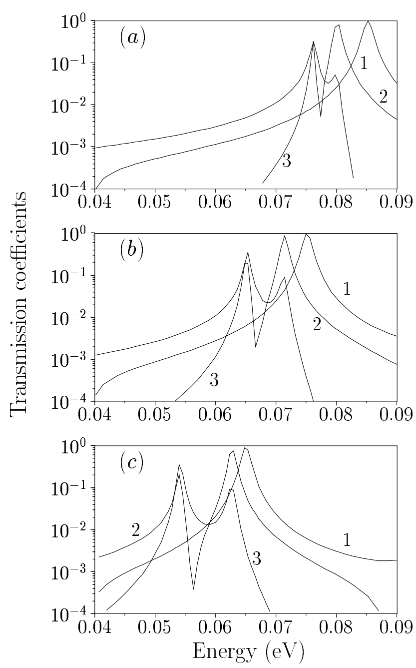

Figure 2. Transmission coefficients versus incident particle energy at $B = 15$ T for the following values of the voltage V: (a) $V = 0.03$ V; (b) $V = 0.05$ V; (c) $V = 0.07$ V.

to the same subband of size quantization. For this reason the dependencies of the tunnelling probability versus energy have two peaks. The positions of different peaks of the tunnelling probability depend significantly on the voltage across the tunnelling structure. The values of the quasibound-state energy of the levels in the well drop with the voltage increasing, which results in the corresponding shifts in the peak positions.

To investigate the magnetic field dependence of the transmission coefficients, we have calculated the functions $T_{kl}(E)$ at $B = 18$ T (see figure 3, where panels (a), (b), (c) correspond to the same values of external bias as in figure 2 and curves 1, 2, 3 correspond to the same tunnelling processes as curves 1, 2, 3 in figure 2, respectively). The cyclotron energy for the states in the quantum well enlarges with the increase of the magnetic field. For this reason the energy separation between maxima of the tunnelling probability increases. Due to great energy separation between the level with $n = 0$ and spin along the magnetic field direction and the level with $n = 1$ and spin opposite to the magnetic field direction in the InAs contact layers, the processes of resonant tunnelling between these levels are forbidden at external bias $V = 0.03$ V. For this reason, curve 3 is not shown in figure 3(a). With the increase of the external bias, the level in the right-hand contact layer with $n = 1$ and spin opposite to the magnetic field direction becomes lower than the quasibound levels in the valence band quantum well, and interband resonant tunnelling into the states with Landau-level index $n = 1$ can occur. The position of each maximum shifts to the left with voltage increasing, because the difference between the energy of the level in the well which corresponds to this maximum and the conduction band edge of the left-hand InAs layer decreases. At some value of the bias, the resonant interband tunnelling through a given quasibound state becomes forbidden. Then the corresponding interband tunnelling current component drops.

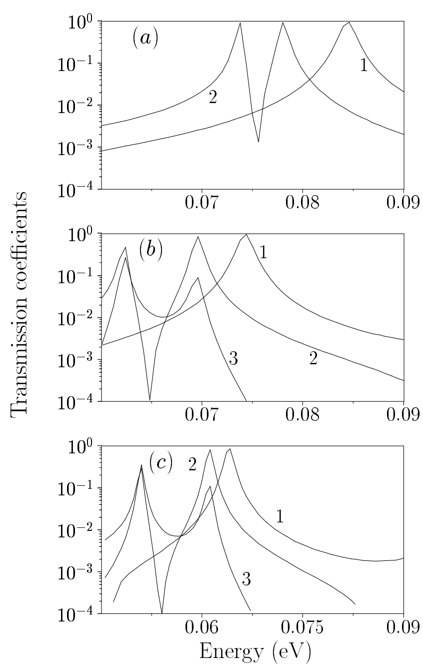

Figure 3. Transmission coefficients versus incident particle energy at $B = 18$ T for the following values of the voltage V: (a) $V = 0.03$ V; (b) $V = 0.05$ V; (c) $V = 0.07$ V.

The calculated I–V characteristics of the InAs/AlGaSb/GaSb RTS considered, with the donor concentration in the InAs contact layers equal to 2×10^{17} cm^{-3} at $B = 15$ T and $B = 18$ T, are shown in figures 4(a) and 4(b), respectively. Curve 1 in each figure corresponds to the total current density. Curves 2 and 3 describe the dependencies of the current-density components on voltage for the transitions with conservation of the Landau-level index between the states with $n = 0$ and spin opposite and along the magnetic field direction, respectively. Curve 4 in each figure corresponds to the spin-flip processes with the Landau-level index changing from 0 to 1. The calculations showed that the contribution to the total current density of the spin-flip processes without Landau-level-index conservation is comparable to the contribution of the processes with Landau-level-index conservation. The dependence of the current-density component on the applied voltage at $B = 18$ T for the transitions between the states with $n = 0$ and spin along the magnetic field direction (curve 3 in figure 4(b)) has two peaks, because the interband tunnelling processes may occur through two quasibound states in the quantum well with different spin orientations and Landau-level indices 0 and 1. For this reason the dependence of the total current density on the applied voltage also has an additional peak. A very small additional peak can also be seen in curve 3 in figure 4(a), calculated at $B = 15$ T. The values of the peak voltage decrease with the magnetic field increasing, because the quasibound levels in the well drop with respect to the Landau levels in the left-hand contact layer. A similar dependence was obtained in the experimental investigations of the I–V characteristics of the GaSb/AlSb/InAs RTS with GaSb contacts [3,4]. We believe that the authors of references [3,4] did not observe the additional peaks on the I–V characteristics caused by the inter-Landau-level interband tunnelling processes for a magnetic field parallel to the current at $B < 15$ T because of the weak mixing of the states of different Landau levels for small values of the magnetic field. The additional peak at 15 T may be associated with the interband tunnelling processes with the Landau-level index changing without scattering. Probably, in the case of an in-plane magnetic field, the inter-Landau-level interband tunnelling processes will also be observable in magnetic fields greater than those utilized in experiments [5, 7].

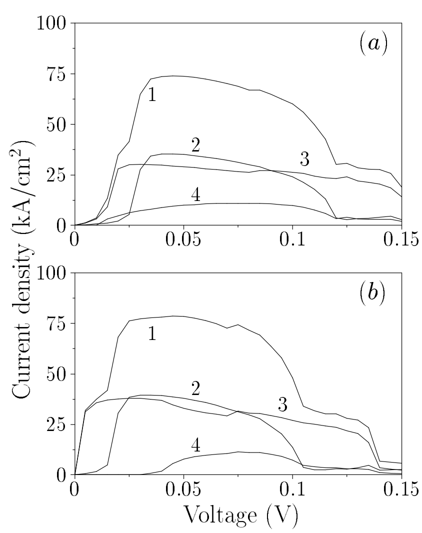

Figure 4. The total interband resonant tunnelling current density and tunnelling current components corresponding to transitions between the states of various Landau levels versus the voltage across the double-barrier structure for the following values of B: (a) $B = 15$ T; (b) $B = 18$ T.

Note that we considered the interband resonant tunnelling in a RTS with InAs contacts and a GaSb quantum well in a magnetic field normal to the interfaces, while this effect was investigated experimentally for a GaSb/AlSb/InAs RTS with GaSb contacts and an InAs quantum well. In the latter case, the GaSb contacts are doped by acceptors and the heavy-hole states should be taken into account to calculate the Fermi levels in the contacts and the interband tunnelling current. With this aim, the influence of higher bands should be considered using perturbation theory, which results in additional terms in the Hamiltonian. In this model, mixing at the interfaces of the electron (light-hole) and heavy-hole states occurs, so the electron from the conduction band state can tunnel not only into the light-hole state but also into the heavy-hole state in the valence band in a RTS made from InAs, AlSb, and GaSb. This results in the additional peaks of the tunnelling probability for nonzero values of the lateral wave vector (see, for example, references [10–12, 17]). Moreover the interband tunnelling through the heavy-hole states in the well influences the observed values of the valley current in the InAs/AlSb/GaSb RTS considerably, as was shown experimentally [6] and theoretically [12, 16]. In the magnetic field, the mixing of the electron (light-hole) states with Landau-level indices n and $n + 1$ and heavy-hole states with Landau-level indices $n - 1$ and $n + 2$ with different spin orientations occurs even in a bulk material [21, 41]. In the six-band model used in references [17, 36], there are four such mixed states for any number $n \geqslant 1$. (The authors of reference [21] describe all of these mixed states using the single Landau-level index n.) In the conduction band, only two of these four states (electron states) have real values of k_z for sufficiently large energies. One of these two states can be approximately characterized by Landau-level index n and spin 1/2; the second state can be approximately characterized by Landau-level index $n + 1$ and spin $-1/2$ in the case of small mixing due to the influence of higher bands. The other states have imaginary wave vectors k_z in the conduction band and correspond to the heavy-hole states. Due to the boundary conditions, mixing of these four states at the interfaces occurs, so an electron from the conduction band state to the left of the InAs/AlGaSb/GaSb RTS with the Landau-level index n and spin 1/2 can tunnel at different values of voltage through two light-hole and two heavy-hole states in the well of each subband of size quantization into the state in the conduction band to the right of the RTS with Landau-level index n and spin 1/2 or into the state with Landau-level index $n + 1$ and spin $-1/2$, which may lead to additional peculiarities of the dependencies $T_{kl}(E)$ and $j(V)$. These interband tunnelling processes will be considered in detail elsewhere.

5. Conclusions

In summary, we have investigated theoretically the interband resonant magnetotunnelling in double-barrier semiconductor heterostructures using the eight-band Kane model. The equations for calculation of the tunnelling current components for the interband transitions from the states with different values of Landau-level index and the total interband tunnelling current density have been derived for the magnetic field normal to the interfaces. The I–V characteristics of the double-barrier interband resonant tunnelling heterostructures have been calculated taking into account the inter-Landau-level transitions. It was shown that the contribution to the total current density of the spin-flip processes with a changing Landau-level index, which can occur without scattering on phonons, impurities, or defects, due to the spin–orbit interaction, is significant. These processes result in the additional peaks of the dependencies of the tunnelling probability on the incident particle energy and on the I–V characteristics of the InAs/AlGaSb/GaSb RTS.

Acknowledgments

This work was supported by the Russian Foundation of Fundamental Research under Grant N97-02-16438a.

Appendix

The set of basis functions used is the following:

$$u_1 = |\mathrm{s}_{1/2,1/2}\rangle = |\mathrm{i}S\uparrow\rangle$$
$$u_2 = |\mathrm{p}_{3/2,1/2}\rangle = (2/3)^{1/2}|p_0\uparrow\rangle + (1/3)^{1/2}|p_+\downarrow\rangle$$
$$u_3 = |\mathrm{p}_{1/2,1/2}\rangle = -(1/3)^{1/2}|p_0\uparrow\rangle + (2/3)^{1/2}|p_+\downarrow\rangle$$
$$u_4 = |\mathrm{p}_{3/2,3/2}\rangle = |p_+\uparrow\rangle$$
$$u_5 = |\mathrm{s}_{1/2,-1/2}\rangle = |\mathrm{i}S\downarrow\rangle$$
$$u_6 = |\mathrm{p}_{3/2,-1/2}\rangle = (2/3)^{1/2}|p_0\downarrow\rangle + (1/3)^{1/2}|p_-\uparrow\rangle$$
$$u_7 = |\mathrm{p}_{1/2,-1/2}\rangle = -(1/3)^{1/2}|p_0\downarrow\rangle + (2/3)^{1/2}|p_-\uparrow\rangle$$
$$u_8 = |\mathrm{p}_{3/2,-3/2}\rangle = |p_-\downarrow\rangle$$

where $|p_0\rangle = \mathrm{i}|Z\rangle$, $|p_\pm\rangle = \mp\mathrm{i}|X \pm \mathrm{i}Y\rangle/\sqrt{2}$, $|\mathrm{s}_{1/2,\pm1/2}\rangle$ are the electron states, $|\mathrm{p}_{3/2,\pm3/2}\rangle$ are the heavy-hole states, $|\mathrm{p}_{3/2,\pm1/2}\rangle$ are the light-hole states, and $|\mathrm{p}_{1/2,\pm1/2}\rangle$ are the states of the split-off band.

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